

An Interview with...

Elena Novaretti
Author of ZoneXplorer and Power Icons

Magnus Johnson talks to Elena Novaretti about her passion for fractals and the Amiga! This is a complimentary extended version of the interview found in Total Amiga issue 24, with additional mathematical discussion.

Fact File

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Elena, you're a prime (pun intended) example of a dedicated Amiga user. How did you become interested in the Amiga, and when?

I spent years of my life with the old c64 when I was younger. I then learned everything one should know about computers and how they work. Getting bored of fighting against its absurd limitations, my natural step forward was the Amiga, a rather common experience. My first Amiga was an A2000 bought in 1991.

There I discovered an unexpected, new world made of windows, icons, multitasking, memory allocations. I felt that the time of hacking the hardware and learning the basics was definitely gone for me, it was time to do something serious and to think of the computer as a real instrument for work, which should help me.

What's your background aside from computers? What kind of education do you have, and what do you do for a living?

I have loved learning things since I was a child. In particular, discovering why things happen and how they work. Thus my interest for science in general. My education stops at high school level. I'm too independent to follow any formal schedule, I like to study only things that I feel attracted to, and to study them by myself. I don't actually have any fixed income, I usually define myself as a private, unpaid artist and researcher.

What other interests do you have besides Amigas and fractals?

Graphics, music composition, photography, mathematics, physics and astrophysics. I also like writing. I'm actually working on a secret project concerning a dream I always had: creating a real autostereoscopic display not requiring infinite information.

What kind of music do you write?

Mainly soft songs, pop, dance, electronic and piano music too.

Are you inspired by any bands/artists in particular?

No, I'm inspired by what comes to my mind only :) I'm very atypical in that: I don't enjoy listening to other artists' music, I get annoyed! And if it's good music, I think "I would like to have written it !!!".

Is music something you want to do professionally, or is it mainly a hobby?

It could have been my first job, because I discovered this talent when I first happened to play something, at primary school. But I cannot manage to organize myself the right way to do it professionally yet. It's hard to explain... even if I have very good composing capabilities, I find it more handy and feasible to start a graphic or code project, or whatever, and finish it, rather than arranging and finishing a piece of music and making it ready for listening... I still have too much music in my head that should come out.

Is any of your music available for listening on-line?

Unfortunately not, but I don't think I will chose the Internet as a way to publish my music.

In the Amiga community, you are known as an artist and a developer. Could you tell us a bit about the applications you have created?



I don't spend a day without writing a line of code, but that's usually just for personal experiments and research. In the past it happened that I coded a couple of thingies that could be useful for other people too, so I decided to release them, for free. Namely, ZoneXplorer and the popular patch PowerIcons. Also I would like to (and really could) write a good paint/graphics program for the Amiga, if I only had the time...

For how long has ZoneXplorer been in development, and what made you decide to start working on it?

ZoneXplorer has a very long story! I started working on it around late 1995 because I needed a tool allowing me to explore any kind of formula on the [x,y] plane and to study custom rendering techniques. What started as a very simple, basic program, grew up and regularly acquired more and more functions, to suit my own needs. As time went by, it was rewritten from scratch at least three times until it got the actual "skeleton" and its modern, flexible design.

ZoneXplorer wasn't intended for distribution however, until many of my friends and fans started asking me to release it, so I was convinced that my tool could

perhaps be of some use and interest for other people as well.

In a few months time I gave it a nice GUI, a high resolution rendering engine, a robust and reliable multitasking design and checked everything to be stable, usable and ready for distribution. I also had to write a very simple manual, check which modules (i.e. formulas) and zones (i.e. locations) were worth including with the official distribution, and also had to prevent reverse engineering on the formulas by implementing a complex (but not fool-proof) encryption method. I have always been very proud of my formulas even if perhaps now many other people have reached the same or similar results.

Then it was finally released to the public for free (but NOT open source, let's not ask too much ;)

Are there any differences between ZoneXplorer and other fractal generators (on the Amiga or other platforms)?

ZoneXplorer has two strong advantages. First, its fast, intuitive and responsive navigation engine, which lets the users move interactively on the plane and easily select the main parameters with just a few clicks, to fully enjoy and explore new formulas in one or more windows, either independently or interconnected.

Second, it is modular, it neither has built-in formulas nor relies on a slow interpreter. Formulas are written in C code which are simply compiled to generate a "module" and then directly loaded and explored.

Anyone can enjoy writing custom modules and explore them following very simple instructions. The necessary environment is included in the distribution and ready to use without assigns or some boring installation.

Also, module writing and compiling will be totally automated and integrated in the main program in the future. Pretty much anything may be written in a module: from the complex fractal generator to the gradient generator for computer graphics purposes or even a simple code to display $y=f(x)$ or $f(x,y)=0$ equations.

In the Amiga market there doesn't actually exist anything even remotely similar. On other platforms I don't know, because I mainly use Amiga, but I heard of a program called Ultra Fractals for PC which seems a good rival to ZoneXplorer (but it's commercial!).

The only drawback of ZoneXplorer is that I wrote it using Amiga OS 3.5/3.9 ReAction classes, and compiled it for both Amiga OS 3.x 68K and for MorphOS. 68K users may find it very slow unless they use Amithlon or UAE on fast machines. MorphOS users, on the other hand, have to pick up all the needed ReAction classes from an Amiga OS 3.5/3.9 CD to run it, and anyone who doesn't own Amiga OS is forced to download the old (bust still compatible) ClassAct classes from Aminet. I know, that's frustrating, but as I will explain later, my goal would be to port ZoneXplorer to Amiga OS4.

Is it possible to use ZoneXplorer to render just about any fractal equation you could throw at it, or does it have any limitations as to what it can do?

ZoneXplorer was designed to pass on to the formulas the following six parameters: X, Y which are the points on the plane to plot, and four constants A, B, T and the integer IT. By properly using them one can write everything.



For example, a plain module to compute a Julia set would process [A,B] as the complex number C and iterate 'IT' times starting at value [X,Y] which is Z-zero on the plane, perhaps using 'T' as threshold value to control the gradient spreading, or as an upper limit to reach before stopping the iterations.

All the module has to do is to compute these parameters and return a 32-bit ARGB value to colour the requested pixel. By intelligent use of these parameters one can also write a module to generate Lyapunov sets with binary masks... the only limit is the imagination (and, you may think, those six parameters? Well, I really never felt this was a limit, six seems okay for most common and uncommon applications...)

ZoneXplorer may of course change the way those parameters are sent to the module (for example passing the point on the plane in [A,B] and the C constant in [x,y], in both, or vice versa), to switch on the fly between Mandelbrot-like and Julia-like representations of the same formula.

How come it isn't possible to enter formulas for "rendering" on the fly? I mean, isn't fractal geometry complicated enough as it is without having to know how to code?

If a fractal program comes with an interpreter it will still require using its own language, its own syntax. Using C requires no particular knowledge of proprietary syntax or language, since C is the most popular, lowest acceptable level language. Also, using C allows someone who writes a module to make virtually **everything** inside

a module, not only writing a formula in a string gadget like $Z=C^n+P$.

You can handle arrays, use subroutines (functions), and create any algorithm to compute the returned colour value. Using C gives an infinite flexibility to the module writer. I would point out that no OS know-how is required at all to write modules, it's really simple and well documented, and there's a default #include file containing all the needed macros, to hide everything you don't need or don't want to know.

Examples are included as well, and even someone who doesn't know anything about the C language can really play around, writing their first module with minimal effort. Guaranteed!

Obviously, anyone who stumbles across a fractal would probably find them very fascinating, but was there anything specific about them that attracted you to the concept? And when did you first become interested in maths and fractals?

I discovered fractals the first time in about 1994, playing with an old PD Amiga program. At the beginning I couldn't imagine the whole truth behind them, I just thought they were generated by some ugly and complex algorithm. When I discovered their true essence, the fact that the program doesn't actually create them but simply displays them (and note the importance of these words), then I became fascinated.

I then searched and learned any information available about fractals, and a new universe was disclosed before my eyes. Maths

too, which I knew enough, but never thought of as more than a useful tool, suddenly turned into the image of God, or almost. I understood that fractals do exist a priori. A concept hard to assimilate and to approve of for everybody, I admit.

What I could not suffer was the poor methods used in those times to display them: bands, whereas a continuous gradient would have been more expectable and gratifying to the eyes. So I started my challenge: understanding them and working to display them properly in all their brightness.

Is it true that you have no formal education in the field of mathematics at all? Are you completely self-taught?

Yes. That's a choice but also a peculiarity of mine: I must explore things by myself, make experiments, move where I want, break my head on things to understand and accept them.

Wow! This stuff is, after all, pretty complicated, so how did you go about obtaining all the knowledge you have?

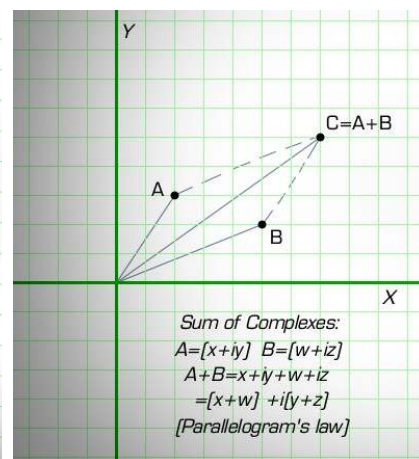
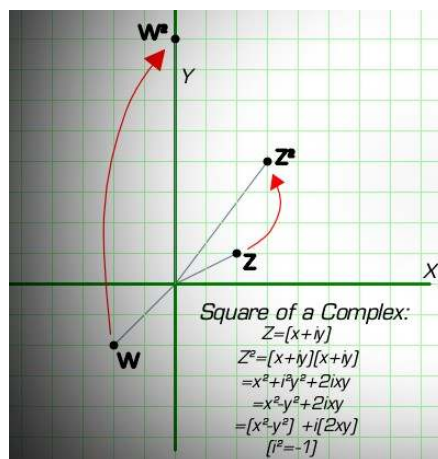
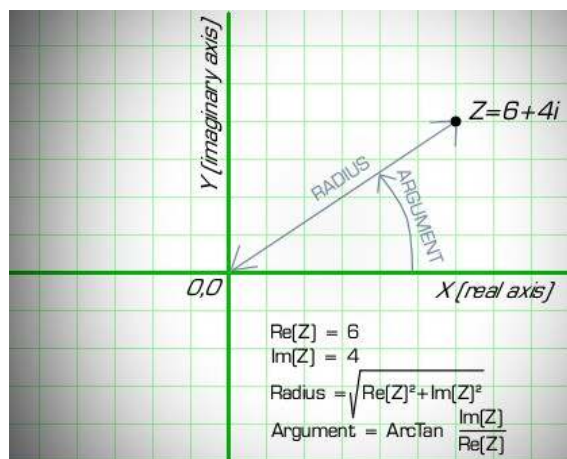
Passion, motivation, patience and much time, that's all!

"Complicated" is, however, a very relative word. Maths itself may appear complicated when studied as crude formulas. If you understand the same things by yourself through motivation and interest, you'll end up with the same concepts and they will appear more fascinating than frustrating to you. Also, one is free to stop at any point he thinks relevant.

There's often no need to learn every name, theory or aspect if you think you don't need it. There's no exam to sit for, if you don't need any piece of paper. There's only some good thing to discover.

Personally, I find Wikipedia to be a real gold mine when it comes to maths articles. Do you have any pointers for anyone interested in learning more about various aspects of fractal mathematics?

When I first started with fractals, the Internet was not as widespread, so I had no connection. Most of the work was made in those times, so I made my calculations and experiments based on almost no books or other resources.



Later I bought some books just out of curiosity. There are many, but all of them treat the subject in a too crude and complicated way, even if they are interesting and useful to integrate into your knowledge. None of them will tell you how to obtain good fractals, of course.

If you search for an old, but still good, book to learn fractal basics with in a widely understandable language, I strongly suggest "Chaos, Fractals and Dynamics - Computer Experiments in Mathematics" by Robert Devaney, Addison-Wesley Publishing.

Another interesting book, a bit funny but covering useful things like IFS (Iterated function systems) and geometrical transformations, is "Fractal Graphics for Windows" by Dick Oliver and Daniel Hoviss, SAMS 1994 (it's enough without reading all the Windows parts!).

And, if you want to get more technical, there's still the Holy Grail "The beauty of fractals" by Peitgen and Richter, Springer-Verlag 1986.

I don't know of any more recent books, sorry. And I have no links to share, there are too many. Just search Google for Fractals and you will understand why I decided to almost stop with fractal art today!

Okay, let's get technical! The fractal known as the Mandelbrot fractal, or the M-set, which appears to be very complex (pun *not* intended) is actually based on the seemingly simple formula $z \rightarrow z^2 + c$. Could you start by explaining how this very simple equation can render such an elaborate graphical representation?

An exhaustive (both philosophical and mathematical)

treatment wouldn't find enough space here. I would just point out that what is actually intended as a simple formula, like $z \rightarrow z^2 + c$, is no longer as simple when iterated.

Let's assume we start with $z=0$. The first iteration gives c , the second iteration gives c^2+c , then $(c^2+c)^2+c = c^4 + 2c^3 + c^2 + c$, and then the previous polynomial squared plus c . You see, in general you cannot write the n -th step as a fixed-length polynomial in a function of n .

Indeed, if $c=0$ (or there's no c at all) the sequence simply becomes (formally) $z^2, z^4, z^8, z^{16}, \dots, z^{2^n}$: you can compute the n -th iteration value just by raising two powers.

You cannot predict the n -th iterate otherwise without computing all n steps. And, in fact, you may verify that with $C=0$ there's no fractal, no chaotic behaviour, nothing and nothing, only a plain limit-circle with unitary radius.

Play with other formulas too to verify the same concept. May be a hint?

Could you maybe also explain what the equation actually means? What does "z" and "c" actually represent?

Mathematically, Z and C are complex numbers.

A complex number is a two-dimensional vector which may be defined using cartesian coordinates (x, y , respectively horizontal and vertical component) or polar ones (a radius R and an angle α).

In short, they identify nothing more than a point. Just like a real number (the numbers you normally use) is a point on the line, a complex number is a point on the plane. And a hyper-complex number (or three-

dimensional vector) is a point in the space, and so on.

Z represents both the horizontal and the vertical axis, how can you get a position in a two-dimensional plane from just one value?

You said it, Z keeps both the horizontal and the vertical position of a point.

Z is made of two numbers, a real (representing the horizontal position) and an imaginary one (representing the vertical position). Namely: $Z = x + iy$

with $i = \sqrt{-1}$, called the imaginary unit.

Maybe I'm a bit dense, but how do you "extract" the real and imaginary parts from Z ? I mean, if Z is assigned a value just like any variable is, how do you obtain two numbers from just that one variable?

There's no need to extract anything from Z since you know its value. As with any real variable, you may assign any value to it, with the difference that it's two-dimensional so you must assign to it any *pair* of coordinates (it is nothing more than a point on the plane).

Z doesn't have one unique value but two, a horizontal and a vertical component (or a radius and angle if you prefer). Algebraically, Z is $x + iy$, i.e. both x and y (where i is the square root of -1).

If the imaginary (vertical) component, y , is zero, then Z is a plain real number lying on the x axis. Again, that's plain analysis, I don't think it's for me to explain such simple basics here.

So this formula renders an allegedly infinite creation, with new details appearing however much you zoom in?

Apparently. It's better to say:

detail is potentially infinite, but information is minimal so you won't find any new "things". You'll always find the same intrinsic geometry, the same patterns, but re-combined. All the apparent information does not exist, what comes out is always a progressive reproduction of the initial rule applied over and over on itself.

Are all fractals infinite?

If you with "infinite" mean what I wrote above, yes. You will obviously need more and more precision and to increment the number of iterations as you zoom deeper to see more details. Virtually, you'll never end with a final, indivisible "atom".

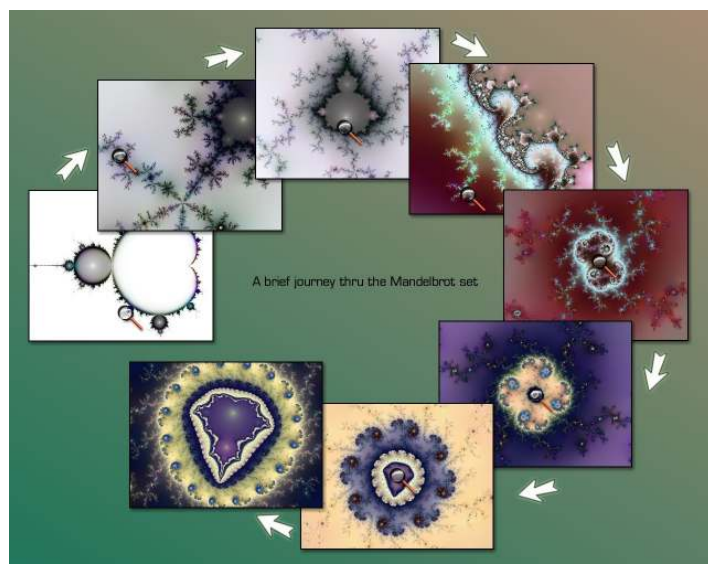
So you can actually zoom in to infinity, really, really infinity, not just a figure of speech, on any given fractal?

Theoretically yes. In practice: for as long as the computer precision used allows you to do so. There are programs using variable-length precision (even if they're usually limited to few, hardcoded formulas): more and more digits are used as soon as they're needed. But no hardware FPU acceleration can be of any help here, and every operation becomes slower and slower as the number of digits increase. I don't think that's so useful, at least, using the actual formulas...

Do fractals extend infinitely if you zoom out as well?

This depends on the formula. Typical Mandelbrot and Julia sets for the family of formulas $z \rightarrow z^n + c$ are limited outside. Other formulas containing inversions or periodical functions may not be limited neither inside nor outside.

Let's talk some more about the M-set. First off, could you explain in a simple way what a "set" is?



In mathematics, a "set" is a finite or infinite whole of numbers (or other things) which can be defined in some way, which you can identify with a common rule.

I can say, for example, that a circle is the set of all points on the plane equally far from a given point (the centre).

One may also define the set of all positive integers, of all prime numbers, of all rational points of a function, et cetera.

Only values between -2 and 0.25 on the X-axis actually belong to the M-set, but how do we know that? Since all the information we need to calculate the M-set is in the $z \rightarrow z^2 + c$ equation, where does the range of this set get defined?

That's a specific matter relative to $Z^2 + C$. Any formula has its specific facts which should be analysed separately. The explanation is long, technical and boring, so I kindly ask you to save me all this trouble ;) The books listed above will explain this to the interested reader.

One should analyse the dynamic of $Z \rightarrow Z^2 + C$ on the real axis (so $x \rightarrow x^2 + a$), understand the so called fixed points which are solutions to $x = f(x)$ and demonstrate for which values of C they exist, are attractive or repulsive, and so on...

The actual fractal is nothing more than the big black blob in the middle, right?

The "actual fractal" is all of the black body, including the main cardioid and all the infinite circles connected to it. I'm following your definition of "black" because the most common rendering

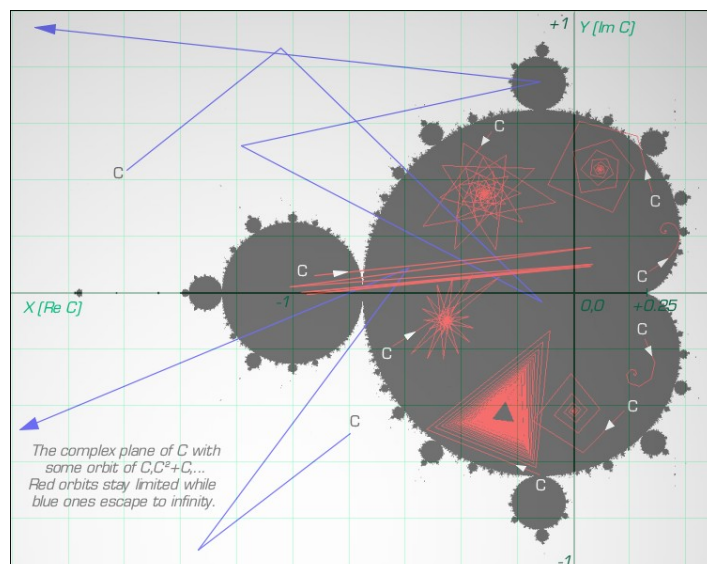
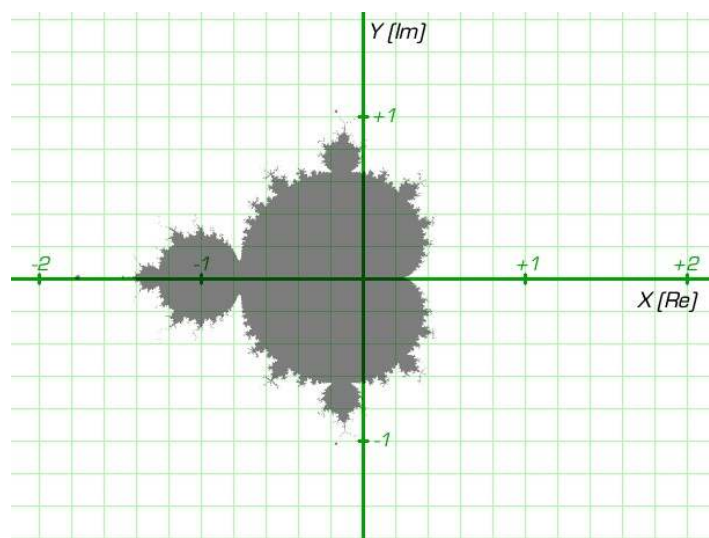
algorithms show it as just black.

Please note, that the definition for the Mandelbrot Set is "all those complex numbers C for which the iteration $Z \rightarrow Z^2 + C$ never goes to infinity" (starting with $Z=0$ or $Z=C$). This is true for all the points usually marked in black - that is the M set. (Well, the true definition of it would be a bit more complex here, so let's skip it...)

So in other words, the fractal itself is pretty boring, it's the "leftovers" that make it become art?

Well, it's not boring at all even if computed crudely in black and white (just assigning zero or one to the points belonging to the set or not, or vice versa) because as soon as you zoom you always find new details and new connections, with infinite little clones of the main figure oriented and placed in every possible way, every one with all the sub-connections of the original, and to infinity.

On the other hand, when



Mandelbrot obtained the first image of "his" set at the computer, it was just black and white but anything but boring... impressive indeed!

But where do all the colours surrounding the fractal come from?

A detailed explanation is needed here. You may imagine painting every person in the world as a black or white pixel if they are male or female.

This may be a first, crude classification to trace some graph. But humans are not only men or women, they're also tall, short, heavy, thin, beautiful, ugly, good, bad, their eyes and hair may have different colours ... so your graph may become more interesting and detailed if taking further parameters into account when assigning a coloured pixel to them.

The same thing applies when you decide to represent a set of points which, upon a dynamic process of transformation (our iterations, for example), may

behave in many different ways: they may tend to be attracted by a single point, or by a fixed-length loop, or even by a fractal loop, or grow indefinitely.

So you suddenly see there's quite a lot of information you can extract from a dynamic process to colour a point depending on what it will do. The oldest, more diffused and simple method to colour a fractal set, is assigning a colour to a point depending on how many iterations it will take to get sufficiently close to a theoretical infinity or to a known fixed point (if the formula allows such attractors).

The first ugly thing coming out is that, this being a discrete value by definition, colours won't be homogeneously spread, but appear as ugly bands. Also, too much information is lost since we take into account only the "escape time" of a point.

Having said that, one could extrapolate a lot of fancy methods to extract colours from the process. Many methods will be coherent and meaningful, while many others absolutely arbitrary and incoherent (just fancy).

Good methods include calculating the continuous potential of a point with respect to infinity or to a finite attractor, that requires some more know how but at least gives continuous spreads. Or assigning a colour depending on the length of the loop a point gets attracted into, and an intensity meaningful of the continuous potential with respect to that loop. Bad methods will give "phantom" elements and are just not meaningful to the chaotic dynamic taking place. Those are,

for example, extracting colour information from separate x and y components at some point, composing the number of iterations taken to reach infinity with the modulus or argument value at that point, et cetera. The Internet is full of all these imbecilities, but some people seem to like them...

Ah, those are two items that keep popping up with complex numbers and Argand diagrams, "modulus" and "argument." Just really quick, what are those?

On a plane, you can address a point using cartesian coordinates (x,y) or polar ones: modulus and argument (read: radius and angle). The radius is the absolute distance of that point from the center (0,0) and is $\sqrt{x^2+y^2}$ while the argument is the angle formed (by convention and counter-clockwise) by the radius with respect to the positive x axis (see the top left image on page 3).

For example, $Z=1+1i$ has $\text{radius}=1.4142135\dots$ and $\text{argument}=45 \text{ degrees } (PI/4)$.

Thus, any complex Z lying on the x axis is a real number with null imaginary part ($i=0$) and may be positive ($\text{arg}=0$) or negative ($\text{arg}=180 \text{ degrees } = PI$).

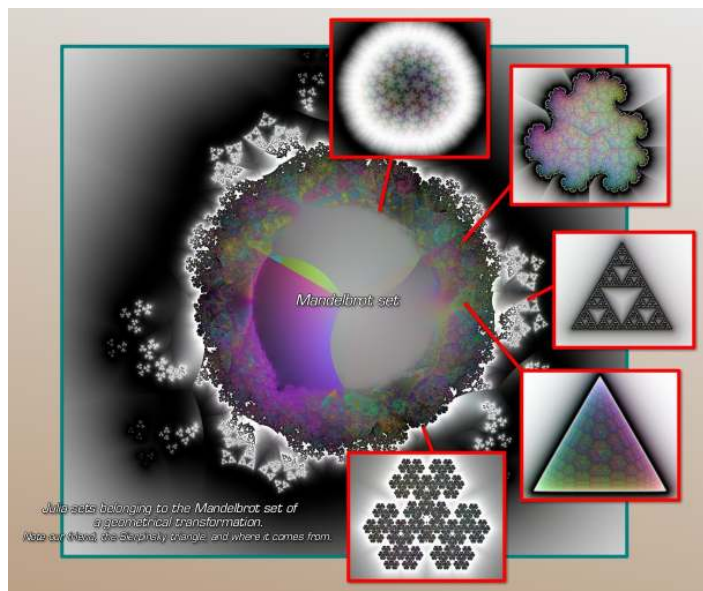
Any Z lying on the y axis has null real part and is called "pure imaginary" ($Z=0+yi$).

As far as "art" is concerned, is it really accurate to label fractals as this? Some people might argue that you need an instrument or a paintbrush to create art.

That's not the point, since you can create art with **any** instrument. With fractals, the computer together with the fractal exploration program and the algorithm used to display the underlying formula **are** the instrument.

Fractal art, intended as I do, is art the exact way photography is. The formula is the key to the world where you plan to do your report. You search for an interesting subject, try to frame it to give the best expression of its essence, then shoot. You may set many parameters to make the subject more expressive, or to make such an apparently banal particular. Really, much like photography.

The M-set cardioid isn't really



limited to what we can see at first glance (even if you take into account the unlimited amount of fragmented detail along the edge as you zoom into it), since there are miniature recurrences of its geometric shape scattered around the "main" fractal itself. How can this be?

Since, as I told above, a fractal brings out no more information than is contained in the basic transformation process iteratively applied (the formula) it's normal for it to be constituted of infinite parts of itself.

How does the concept of a "set" relate to other fractals? If we look at the Sierpinski triangle for instance, which starts out with a given amount of data and then has information removed from it over and over again to create a fractal. How is the "set" defined in this case?

Just as what remains after (theoretically) finishing the process. Sierpinski's triangle may be thought of as a two-dimensional analogous of Cantor's set.

Cantor's set is what remains if you take a line, cut away the central third, and proceed recursively for every remaining third, to infinity. The "dust" obtained is an object made of infinite points, but it doesn't have zero dimensions and it's not even a line: it has a fractal (read: non integer) dimension between zero and one.

Since the Sierpinski triangle gets infinitely reduced, how come it doesn't disappear altogether?

The same reason why you can still see a point even if it is without dimension, or a line, which shouldn't have any thickness by definition!

While we're on the subject of Sierpinski, I just found out that the Sierpinski triangle is present as a pattern in Pascal's triangle. First of all, how weird is that? Is there some profound logic behind this "coincidence," or is it just a fluke that things just happened to turn out like that?

As I have mentioned elsewhere, fractals can be the result of many kinds of, even apparently different, procedures. To construct Pascal's triangle you simply start with a number (1) and then proceed adding lines, whose elements are the sum of the two adjacent elements from the upper line:

```
(0) 1 (0)
(0) 1 1 (0)
(0) 1 2 1 (0)
    1 3 3 1
    1 4 6 4 1
    . . . . .
```

Technically, that's similar to an iterative process where the next result is computed upon the previous state and so on: $\text{Line}_{n+1} = F(\text{line}_n)$, where F here is some function that substitutes any element in $\text{Line}[i]$ with the sum of two adjacent elements. There're many similar methods to compute "cascade" fractals, always starting from a line composed of binary or real cells, then applying some rule to compute the next line starting from the previous one (a rule may be to add two adjacent elements like in the above

example, or computing some binary operator over two or more adjacent cells, etc.). We can then extract the information the way we like (please see my answer about coloring schemes).

One choice may be, for example, checking the disparity of the generated numbers. The curious thing: you will almost always end up with Sierpinski-like triangles!

I never studied in depth the exact mechanic of that, but surely it's a symptom that structures like Sierpinski triangles are quite stable and persistent.

Are there other areas where fractal geometry overlaps "regular" maths in similar ways to the Sierpinski/Pascal relationship?

There're surely infinite cases even if I don't have an example ready now.

It's talking of "regular" which could issue some consideration here... is building a Pascal's tower a "regular" math proceeding? I see it more like a logical, or geometrical play. I could answer with a more abstract but deep concern indeed.

Fractals build themselves starting from very little, or no information, always forming the next stage upon the previous one, thus generating structures. Maths' logic has the same behavior: every operator, every branch builds itself upon the previous one generating even more intriguing and apparently complex schemes.

Are there fractals that don't require the use of imaginary numbers?

Nothing necessarily binds fractals to imaginary numbers but there are some reasons why they're more beautiful and evocative when made with complex numbers, because on the 2D plane any operation on them is 2D-coherent: a complex formula implicitly operates a coherent transformation of both the horizontal and vertical component accordingly.

Other 2D, not strictly complex mathematical proceedings may be iterated on the plane with interesting results, as for example the iteration of symmetrical systems like $x \rightarrow Fx(y); y \rightarrow Fy(x) \dots$

That's a lot of x's, y's and F's you've got there...and arrows, a bunch of arrows too...could you explain that equation chain in "simple English"?

I usually write and think of $x=F(x)$ as a function taking a value x and updating it with the new value, just because in many programming languages (incl. C) $x=F(x)$ means just that, it's not an equation.

But the correct mathematical syntax for that is $x \rightarrow F(x)$, i.e. x becomes $F(x)$. If there're parameters you may write $F_n(x)$, i.e. x^2+n is $F_n(x)$, or $F_{ab}(x)$ i.e. $ax+bx^2$.

Okay, getting back on-topic, maybe it'd be a good idea if you also told us what an imaginary number is?

That's plain analysis. As I told at the beginning, a complex number is a two-dimensional number, a vector, indicating a point on the plane just like real numbers represent a point on the line.

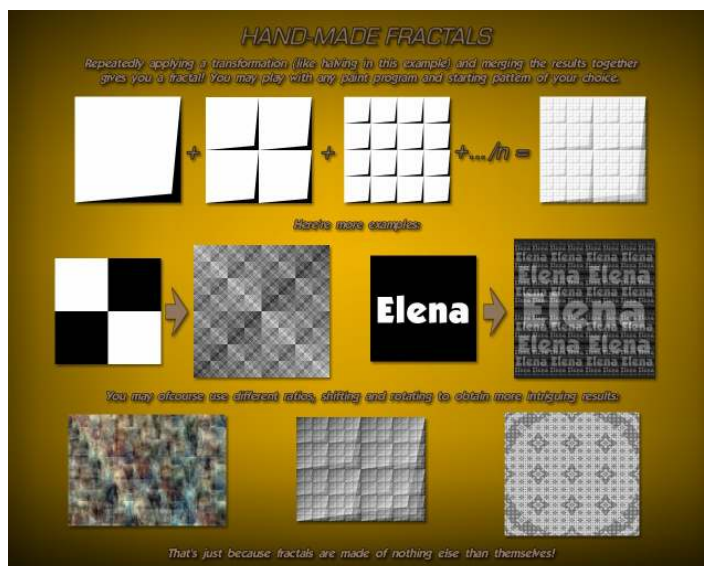
Complex numbers thus have two components, they may be written in the cartesian form $[x+iy]$ or using polar coordinates $[radius, angle]$. Both methods are valid and mean the same thing.

In order to acquire meaningfulness as a complex number, when using cartesian coordinates we must consider the imaginary unit "i" which is the square root of -1 ($i^2 = -1$). The real and imaginary parts are plotted on the horizontal (x) and vertical (y) axis respectively. So $x+iy$ must be thought of not as a plain, intuitive addition but as the horizontal and vertical coordinates of our point, our complex number, usually called Z .

"i" must also be thought of as a 90 degree counter-clockwise rotation:
 $i^2 = -1$, $i^3 = -i$, $i^4 = 1$.

Turning a complex number expressed in polar coordinates (angle Alpha and radius R) to cartesian coordinates is simply done with $Z = R(\cos(\text{Alpha}) + i \sin(\text{Alpha}))$.

Adding two complex numbers Z and W gives a number Q which has the sum of their real parts as the real part and the sum of their imaginary parts as the imaginary part; $Z+W = (\text{Re}Z + \text{Re}W) + (i\text{Im}Z + i\text{Im}W) = Q$.



It may be seen as the fourth vertex of a parallelogram having the remaining three vertices in Z, W and in the origin $[0,0]$ (parallelogram law). The sum of two complex numbers as a direct function of their radii and angles is unfortunately not allowed by mathematics (God said "NO") but that's another story...

The product of two complex numbers is a complex number having the sum of their angles as the angle and the product of their radii as the radius. Thus, raising a complex number to a power n gives a complex number with the angle multiplied by n and the radius raised to n . Thinking of them geometrically may be more useful to understand what really happens within complex calculations (see the images on page 3 for illustrations of complex numbers).

So working with complex numbers is a lot like working with vectors in a real xy-plane?

Complex numbers are vectors you can treat algebraically.

You wrote that 'God said "NO"', now, fractals (the Mandelbrot set specifically) are sometimes (half-jokingly) referred to as "The fingerprints of God."

Likely, just as squares and circles may be thought of as the fingerprints of Evil!

Do you believe in God and, if so, do you think there might be sense to that statement?

I don't believe in God the way most people do, that's for sure. I can accept God as a concept of everything, the whole, the perfection. Far from having a

face, a voice, a son and a wife, being material and being able to interact with our universe, breaking the unbreakable cause-effect law with miracles.

Back to the maths, are there fractals that can be plotted in a three dimensional xyz-space as well? And, what would such an equation look like?

That's another interesting question needing an exhaustive answer.

In short, not the usual way. Intuitively, we could use three-dimensional numbers and operate the same way we talked about as for two-dimensional numbers (complex numbers). In fact, what comes out doing so is not what we would expect, but perhaps that's because we often expect the wrong things.

Having said that, to make the same thing in three dimensions we need numbers with three dimensions in order to obtain a coherent image. We could express such a "hyper-complex" number adding one more imaginary coefficient, we'll call it "j". If "i" represents a rotation on the Z axis, we can think "j" as a rotation in the third dimension on the Y axis (on the X axis would be another valid alternative, but we must make a choice. And, believe me, when in mathematics one has to make a choice it's a symptom that something is going wrong).

Well, so $j^2 = -1$, a point on the Z axis perpendicular to our plane in front of our nose. $j^3 = -j$, another 90 degree rotation around the Y axis. So j is another square root of -1, and that sounds strange too because a fundamental math theory states that an nth-root

always have n values. But let's proceed all the same.

$j^4 = 1$, the point at the opposite side of j , and that's fine, and $j^5 = j$ returns to 1 again. The big issue arises when we ask: and what's j^i ? Following the same logic, that should be a rotation of i on the Y axis, which gives again i , or a rotation of j on the Z axis, which gives again j .

So, what's j^i ? Is it j or is it i ? We must assume that the first term is the rotator, and the second the rotating, or vice versa. In the first case we'll get $j^i = i$ and $i^j = j$ while in the second case $j^i = j$ and $i^j = i$. In any case we break the commutative law for which a^b must be b^a !

The resulting image, if computed, appears incoherent and discontinuous, or at least, anything but beautiful. There's perhaps some obscure mathematic force forbidding the existence of numbers with more than two dimensions observing the commutative law. Somebody also obtained pictures of slices of four-dimensionals Mandelbrot sets:

they perhaps have a beauty we cannot appreciate, or simply they're just nonsense. There's still room to investigate this field.

But fractals can be obtained with many other methods than iterating plain analytical functions, like applying geometrical transformations over and over. Thus, geometrical fractals can be created with any number of dimensions. You can make Sierpinski pyramids or tetrahedrons, sponge-like cubes, balls attached to smaller balls,...

Could you explain to me, reeeally slooowly, what all that math means?

Following the discussion above, try playing with the usual, simple case, Z^2+C with Z and C being hypercomplex numbers,

$$Z = x+iy+jz$$

$$C = a+ib+jc$$

so:

$$Z^2+C = (x+iy+jz)^2+a+ib+jc = x^2-y^2-z^2+2ixy+2iyz \text{ (or } 2jyz) + 2jxz + a+ib+jc =$$

$$x \rightarrow (x^2-y^2-z^2)+a \text{ (real part)}$$

$$y \rightarrow 2xy+2yz+b \text{ (imaginary part i)}$$

$$z \rightarrow 2xz+c \text{ (imaginary part j)}$$

....assuming $i^2=j$, or:

$x \rightarrow (x^2 - y^2 - z^2) + a$ (real part)

$y \rightarrow 2xy + b$ (imaginary part i)

$z \rightarrow 2xz + 2yz + c$ (imaginary part j)

....assuming $i*j=j$.

Compute it as you would with a plain Mandelbrot or Julia set, only you're working in 3 dimensions so you must choose a 2d slice to work on. Iterate x, y, z until the radius $\sqrt{x^2 + y^2 + z^2}$ exceeds an arbitrarily high value.

Then witness the ugly pictures coming out.

So is it sensible to have an imaginary vertical axis in a 3D-fractal, or do 3D-fractals only work with real numbers? And what would a "complex room" look like?

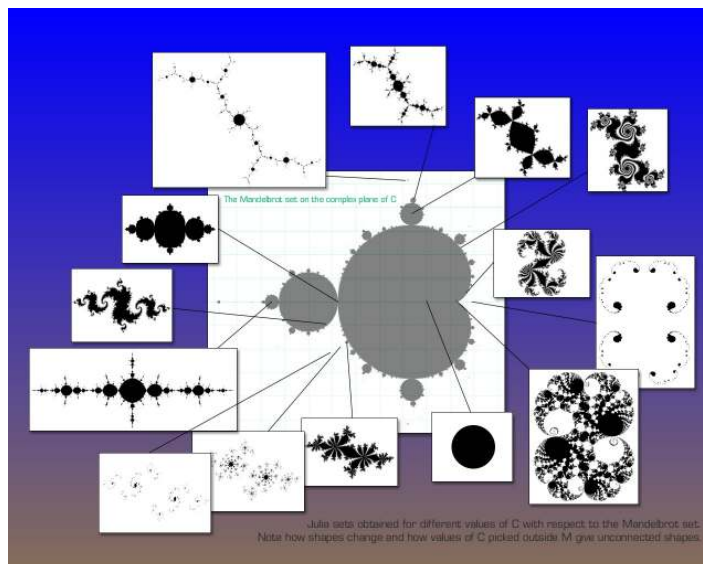
There is no sense in thinking of a space with some complex dimensions and some real ones. An n -dimensional space will have n real dimensions, and (theoretically) the numbers which best fit it are n -dimensional numbers or vectors. You might however think of some kind of 4-D space when dealing with complex functions in the form $Z=F(W)$: there, for any (2D) value of W there is one or more corresponding (2D) value of Z . So a complex equation could be imagined in a kind of 4D space, but it's more correct to say that it's a space with two complex dimensions.

The M-set is based on what is known as the Julia set, but when you look at the fractals they produce they look very different from each other. How can this be if they are based on similar formulas?

Let's explain the relationship between an M-type set and a J-type set for the same given formula. All that applies only to formulas using complex numbers, with one iteration variable and one constant (eg. Z and C).

Technically, only two ways to represent the process on the complex plane are possible: on the plane of C (setting a proper and constant value for Z -zero) or on the plane of Z -zero (with C constant and freely selectable). For any formula of the described type (namely $Z \rightarrow F_c(Z)$), we're used to refer to as a Mandelbrot-type map (the former) and as a Julia-type map (the latter).

It comes out quite intuitive that the former method gives a kind



numbers, but writing down the final C code by hand results in a more optimised code and then in a faster module). But when I load the new module for the first time it's always a special feeling!

What ways are there to create fractals? I'm guessing you could plot just about anything that resembles an iterative function, but are there any general guidelines one could keep in mind regarding the manipulation of the equation, to achieve certain results?

First of all, you must know mathematically what you're doing to plan any work. As I said before, there must be a geometrical coherence between the space you plan to work in and the type of numbers used or vectors. You won't obtain anything interesting putting random functions, unless that's done with some know-how.

There are many different categories of fractals, what are the differences between them? Maybe you could explain what the descriptions on your old fractal homepage mean?

- Maps of polynomial and rational functions.
- Maps of mathematical functions with geometrical distortions (discontinuities) applied.
- Maps of transcendental functions.
- Pictures from Julia sets of arbitrary geometrical transformations, rather than real math functions, allowing second order discontinuity (jumps).

I tried to distinguish between the core function used because they tend to give different and characteristic results.

The first section is dedicated to polynomial and rational functions. That is, plain algebraic operations like integer powers, products, sums, fractions. $Z^n + C$ falls in this category, the same for $Z^3 + Z^2 + CZ$ or $C/(Z^2 + C)$ or $1/(Z^n + C)$ or Newton-type functions or even longer expressions.

The second section hosts images generated with non-mathematical functions, applied to mathematical ones, or on their own. I refer to "non-mathematical" as those algorithmic operators from the

simple $\text{abs}(x)$ to more complex macros like $\{x = \text{abs}(x) - 2; \text{if}(x < -1) x = -1;\}$ or $\{x = \text{abs}(x) - 1; \text{if}(x < 0) x = -x;\}$, not expressible with a finite mathematical expression; discontinuous because they have junction points but still no "jumps". Also inversions or reflections with respect to a circle or a square - all operations which cannot be obtained with pure, finite mathematical operations.

Third section keeps pictures of fractal sets obtained iterating transcendental functions, mainly exponentials, or even trigonometrics, alone or together with rationals or polynomials. One may be the famous $Z \rightarrow C * \text{Exp}(Z)$ for example. Their shapes are characteristic of infinite-degree polynomials by the fact that any transcendental function can be obtained with an infinite polynomial with infinite degree. If you compute the M or J set for $Z \rightarrow Z^{100} + C$ you already start finding shapes similar to the ones obtained with exponentials.

Finally, the last section contains my very latest work with fractals usually obtained with random procedures without actually using any random stuff (random is an ugly word!). Mathematically, these functions may be thought of as discontinuous functions which **may** have jumps. Geometrically, as the composition of transformations over and over, like shifts, scaling and rotation. There I use an algorithm allowing to explore Julia maps of those fractals commonly known as IFS without, in fact, using any random process but computing those sets pixel by pixel in an ordered way. They're spectacular and give perhaps the most similar pictures to natural phenomena, like ferns, trees, stone patterns, sponges, rocks, sand, fire...

Quite a few of the pieces in the latter category, in my opinion, seem to slightly resemble the Sierpinski triangle and the Sierpinski carpet. Are those fractals really part of the Julia set?

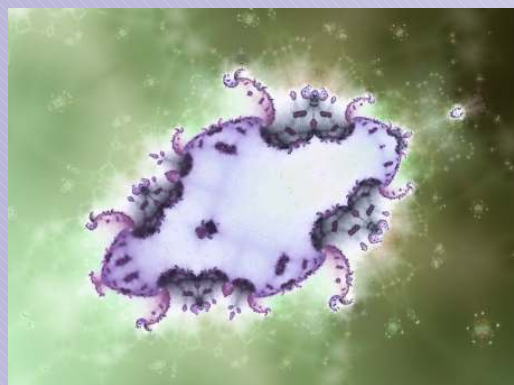
"THE" Julia set usually refers to the Julia-type map of $Z^2 + C$, as I explained before. Those are Julia-type maps of the functions described in the previous answer, computed on the plane of Z -zero with C parametric.

01: Polynomial and Rational Functions



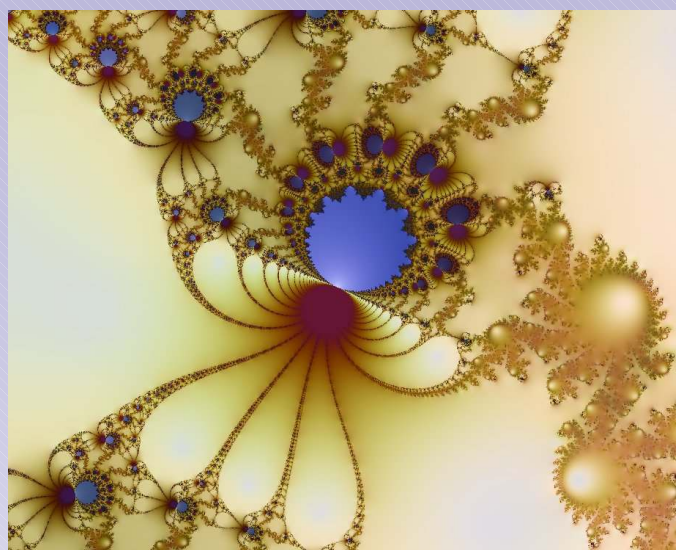
A Mandelbrot "atom" found deeply inside the well known Mandelbrot set for $Z \rightarrow Z^2 + C$, identical in total to the whole set.

02: Non-mathematical Functions



Fractals (mostly from Julia type sets) obtained from discontinuous mathematical proceedings. Monsters here are literally always behind the corner!

03: Transcendent Functions



Exploring exponential fractals you often end up with similar "dipoles", i.e. the eight-shaped objects filling this picture. The blue areas are the corresponding Mandeloids we are used to see in rational fractals, only they have an infinite degree. The iterative dynamic is generally chaotic inside them. I cannot speculate on their opposite red part yet.

04 Discontinuous Functions



It's spectacular how many of these resemble natural surfaces like ground, stones, coral, sponges, ferns, water, crystals and so on.

$F_c(Z)$ here is any iterative geometrical transformation, not expressible with simple, plain mathematic operators like sums or multiplications but still rather simple and, especially, 2D-coherent.

Apparently, there are even fractals based on Sir Isaac Newton's mathematical research. Does everyone who's got any reputability to their name have their own fractals?

Not necessarily. The so called Newton's fractals are based on Newton's formula, which is an iterative process to find solutions to an equation. Newton's formula simply states that, given any $f(x)$ and starting with some initial value for x , the iteration $x \rightarrow f(x) / f'(x)$ more or less quickly converges to one of the solutions to $f(x)=0$.

The truth is that when there are many solutions, the one the proceeding will converge to depends on which initial value you chose for x . In some cases the iterations fall into an endless loop, with fractal shape, surrounding the solutions without ever reaching them.

If we work with complex numbers, so to make the process interesting on the 2D plane, it works the same way and sometimes gives an intriguing dynamic. We chose an arbitrary function $F_c(Z)$ with a parameter C and iterate it on the plane, then returning a meaningful color value depending on how it behaves after a number of iterations.

You may obviously chose to display it as a Mandelbrot-type map (on the C plane with a properly chosen Z -zero) or as a Julia-type map (on the plane of Z -zero picking different values of C). A common coloring scheme is based on checking how many iterations the process takes for any C (or Z -zero) to converge to some known solution, or to any solution.

Have you found your fractal knowledge to be relevant in any way beyond "just" creating art?

After being interested in fractals for art I then discovered how pretty much everything is fractal even if we cannot immediately see that. It can be a real new philosophy to understand maths, logic, everything. Sorry I can't



explain it better.

Every process implying the principle of minimal information iterated over and over may be thought of as fractal. I saw fractal laws when adding waveforms, when thinking of music or even of the function 2^x so diffused in nature: take a stone, add it to itself, add the result and so on.

The universe itself is surely a fractal, and I speculate it should contain no information at all (all we see is only apparent). But if so, some different rule must be applied to every step than the ones we have discussed here.

Magnifying into a Julia-type map (or even into the Sierpinski triangle or similar fractals, which are actually Julia-type maps of non mathematical functions for some given parameter) we have a perception of complexity but we're really always finding the same pattern applied on itself to infinity. Deeply exploring a Mandelbrot-type map already gives a sense of more complexity, because we also have a parameter of the formula changing all around the plane, so shapes locally change; but we'll never end up with a square or a Sierpinski triangle or an octagon or whatever else even zooming billions times into it (okay, we can't actually prove that but it appears quite reasonable). My dream would be to find the formula of the universe!

On the other hand, our universe might only be one of infinite, possible ones with some (or even infinite) key parameters, even if my crude intuition makes me think the first hypothesis is more probable.

Have you figured out the length of Italy's coastline yet?

If the sea stayed still I could try!

Cauliflower, ferns and clouds are sometimes referred to as "natural fractals," are there any other, maybe less obvious examples of where fractals might be seen in nature?

They're everywhere but the big "noise" present often makes them not so appreciable. Cauliflower and some broccoli species are perhaps the most astonishing examples of quasi-perfect fractals in nature.

Also atoms constituting molecules constituting crystals constituting matter constituting planets constituting solar systems constituting star clusters constituting galaxies constituting galaxy clusters constituting super galaxy clusters... there are upper and lower limits but the trend is a big something made of something similar and smaller, down to electrons orbiting around the nucleus like planets around a star, and so on.

But remember, a fractal intended as a perfectly defined mathematical object is only an abstraction, just like it is for the circle: you'll never find a perfect circle in nature, and the same is true for fractals.

But fractal research is currently being used in creating image compression routines, so wouldn't that imply that even natural objects can be described mathematically? At least little bits at a time per formula.

I was speaking of general rules. There's too much noise in nature to reconduct everything to a plain algorithm, only a qualitative approximation is possible. I don't know how fractal compression routines actually work. (It's not

something I'm very interested in).

Have you tried yourself to create anything "realistic" in ZoneExplorer?

It's not possible at this point to build a precise image starting from a formula, it would be more like using a ray tracing program while actually exploring fractals is more like photography in new, (partially?) unexpected worlds.

But perhaps the formulas used in the last section I described earlier represent a new approach toward almost building them. There, you put precise data in some array in your module telling exactly how many transformations will take place at any iteration step, and specifying for each one where the abstract sub-shape should be placed with respect to the parent shape, how much scaled, translated and rotated. No shape actually exists because you start from just a virtual, imaginary shape which can be anything. The real shape is the one you will obtain as the limit for the entire process after many iterations, but it's not difficult to imagine it. I however feel more fascinated by mathematical fractals, because they're not based on artificial transformations and almost nothing is arbitrary.

Fractals are related to a branch of mathematics known as chaos, could you explain briefly what this means?

Inexpert people might think that a process said to generate chaos would generate some incomprehensible pattern of random dots or lines much like an untuned TV screen. The truth is very different: displaying the output in the correct way always uncovers spectacular geometries with their intelligent connections and intrinsic beauty.

Chaos is a deterministic but not expected, apparently disordered and not predictable behavior. Chaos usually arises by iterating nonlinear processes, i.e. continuously applying a non-linear transformation of any kind over the previous result.

Not predictable means that you have a process where you cannot know the value it will assume at the n -th step without computing all n steps, but it's still strongly deterministic: the unpredictable n -th value will always be the same if using the

same process with the same parameters.

In nature, determinism apparently breaks down because of "noise". Since a chaotic process is strongly sensitive to initial conditions, an infinitely little change in them often brings dramatical changes to the next steps. That's what scientists call the "butterfly effect" (which has nothing to do with MorphOS!)

We may think of the universe as a very big system where every state is computed applying some fundamental rules to the previous state: it's the principle of cause-effect. Nothing can happen without a cause, and no cause can happen without another cause. Whereas we can think this rule breaks down is when we're missing something...

Are there any other aspects of mathematics that fascinate you as much as fractals?

Pretty much every branch of mathematics, even if not everybody can see its beauty. I made personal studies on what I call "fourth-order operators", i.e. tetrations, superlogarithms and super-roots. Math is very fascinating: you often expect to see connections made with human logic, while math uses its own logic, it's *the* logic.

Which one of your fractals do you like the most?

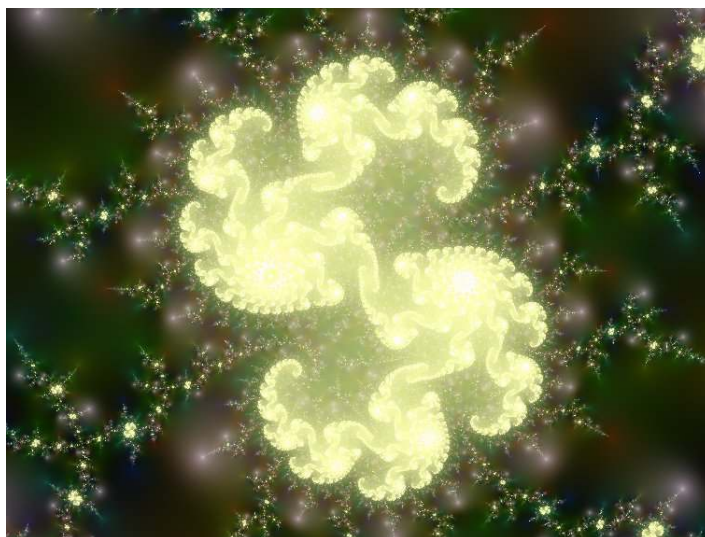
To answer shortly, "Inner World" from section #1 and really many from section #4. I also like many transcendent fractals just because it was a real challenge to me to map the dynamic of such systems, which tend to "explode" so rapidly.

And which one of your formulas do you find most beautiful?

Many formulas used in section #4 gave really unexpected and astonishing, realistic results. Some pictures actually resemble photographs of real things.

Your artwork was on display at the AmiGBG fair in Sweden, what other Amiga fairs have you been involved in?

Well, I showed them for the first time here in Italy at Pianeta Amiga 2002. Then I opened my web site, so there was no longer a need to attend fairs and spend money to print new fractals since few people usually want to buy them. Also many people just



prefer to see me in person rather than my prints!

Do you intend to keep appearing at Amiga fairs, either yourself or through your art?

I'm generally available for any invitation, even if I don't like to travel outside Italy.

If someone wanted to have an Elena fractal on the wall, would he/she have to go to an Amiga event to get hold of one, or are there other ways to purchase your artwork?

Anyone can purchase prints of my artwork directly from me, just e-mail me!

Since a few months back there is a message on your web page stating that you have more or less "abandoned" your fractal gallery, how come? Have you lost your interest in fractals?

Partially yes, even if a big love never dies completely. I unfortunately don't have as much time to spend with them as I did in the past. Also, the Internet has become full of psychedelic fractals during the last years. Yes, some people still understand that mine are different, because I made them with passion and acquired all the necessary maths basis, and developed the software used to suit my own needs, and that many of them are very original because they contain a lot of personal research.

But for the plain visitor that makes no difference. Perhaps he prefers fractals made with fancy rendering schemes or with tricks, processing and compositions. Those are not plain fractals, rather photo compositions... it's

cheating!

Does that mean that there will be no more new fractals from you?

Nothing prevents me from making new fractals if I feel inspired again and/or if some great new ones come out! My fractal gallery will always be available and updated if possible, even on the new site.

And what about the OS4 native version of ZoneXplorer?

Not only is it planned since a long time, but ZoneXplorer will perhaps see its future on Amiga OS4 only. It's a time of uncertainty in the Amiga world, and I'm not very convinced of its future, but for many reasons I think I will switch to Amiga OS4 as soon as there will be modern and powerful hardware to run it.

MorphOS is moving into directions I simply **don't like**. Genesi doesn't support the MorphOS team any more, so... the end of dreams.

Amiga OS4 running on Pegasos would be nice, but too many bad people don't want it to happen, for pseudo-political reasons...

What are you planning to feature on your new website once it goes on-line? And when do you think this will happen?

Well, when I made the old site in early 2003 it was mainly intended to host my fractals, adding just a little information about me. You often start a project to go in one direction, then you realize you're going the opposite way. It was full of rants, allow me to say that. Also, it was focused on Amiga and fractals only.

The new site will be less polemic (times have changed...), more focused on what I really am and what I really do, it will have sections to host different art work, not just fractals (such as photos, computer graphics, renderings,...), interesting tutorials and texts about what the Amiga is (for those who really don't know of it!), about fractals, some research papers on maths and optics and much more.

When will it happen? Sorry, I'm very busy now and cannot give any date yet! What I can say for sure is that it will still be simple, fast, navigable and light as it was before (no flash or Java)!

Do you participate in any Amiga forums?

I'm not one of those people who spend all their time on rants and flames on the forums. I only read the Italian news/forum AmigaPage (www.amigapage.it) and MorphZone just to be up to date, and I write something only when I need help or I really cannot keep quiet! I'm also on some mailing lists.

What Amigas do you own and use these days?

I now have only one "Amiga", which is my PegasosII-G4 with MorphOS. Better having one very good machine to work with than many old relics to play with!

What's your reason for sticking with Amiga systems all these years after the demise of Commodore?

It was just in those years, about 1994-1995, when I learned programming on the Amiga and started my work with fractals, so I stayed with it out of necessity and convenience. I also didn't like the alternatives too much, which in addition were anything but cheap at those times, and not as mature as they are today.

Do you use any other systems today besides your Pegasos, and for what purposes?

I also have two PCs in my lab, both with Windows XP pro. One, a still powerful P4 2.4 GHz, is rack-mounted and used to make music with CubaseSX and many virtual instruments and effects. Another PC equipped with an older AthlonXP 1800 is used for office purposes and to do all things unfortunately impossible or problematic to do with the Amiga, like printing, scanning, navigating some spiteful sites full

of flash or Java. However I prefer to use it from my Pegasos with RDesktop so I notice the smell of Windows a bit less!

How do you think OS 4 and MorphOS compare to other operating systems?

Amiga OS in all its incarnations is an essential but robust and fast operating system compared to the other OSs available. Perhaps too essential, some might say. But its essentiality allows one to know and master it almost completely, to quickly build a project from scratch, and to quickly trace the cause of a problem if one arises.

What do you mainly use your Pegasos for today?

In short: I use my Pegasos to do everything I can without having to resort to a PC :) I write code for my research and experiments, I make graphics, fractals, image processing, write text, navigate the Internet, handle e-mails, watch movies,...

Do you use any scientific Amiga software, maths related or otherwise, that you could recommend?

Really, no!

And perhaps there's no such software available for the Amiga AFAIK. Every time I need some computation I usually write the C code myself to do the job.

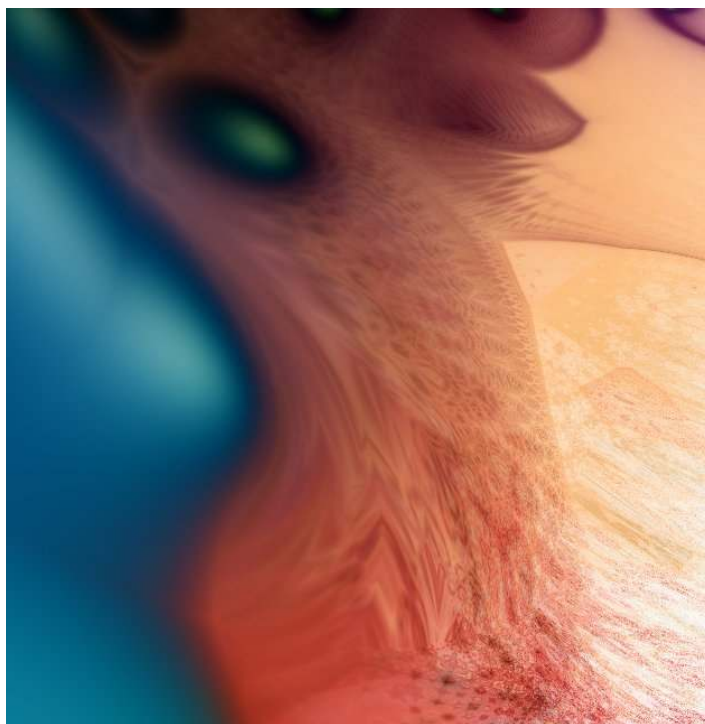
Well, when you really need a gui and an interpreter for very heavy works, there's still the PC with Mathematica, Maple,...

What do you find most exciting about the Amiga today?

That it's still you who command it and not vice versa!!

What do you think of the Amiga's chances for survival?

Since even before the days of



Commodore's bankruptcy, Amiga kept on surviving as a moribund attached to its life-aid machine. It will perhaps never die completely, or at least, not all of a sudden. And that's more frustrating, sad, painful. I saw names, people changing, but the substance and the mentality below is always the same: destructive. The community made of private users and volunteers is that life-aid machine.

The truth in my opinion is that a little computer niche has no chance to survive nowadays, when even more important names die or conform before the Wintel monopoly. Linux survived and will always survive because it's open source, free, runs on standard hardware and there's no real business behind it, and it will be the only alternative to trusted computing (if that should ever happen...)

In our dying little community we have too many bad guys. Instead

of helping one another to survive they are: proud, selfish, pathetic, attached to money, jackals, bellicose or childish! A weak foundation for a stable future, don't you think?

For how long do you think you will keep using Amiga (and MorphOS) systems?

As long as there's some support, both hardware and software, as long as the community won't be completely dead. As long as I can still find IDE HDs/CDs/DVDs or old Radeons or Voodooos without searching on eBay... urgh!

What are your best and worst Amiga related memories respectively?

My happiest memory is when I released PowerIcons: I felt loved for giving something useful. My worst memory is about those bad times around the mid-90s, when we couldn't even use a "normal" graphic card because there was no RTG yet... I was almost

tempted to get a terrifying PC or even a Mac...

What are your plans for the future?

Regarding Amiga public projects everything is frozen at the moment and I cannot predict anything, most depends on the future of the OS and on the available hardware. I will definitely switch to Linux if things on the Amiga side go totally wrong.

At present all my work is focused on stereoscopy and autostereoscopic displays. Not very related to Amiga, aside from the fact that I use it for 90% of things ;) I will of course keep on working on music hoping to be able to publish something in a reasonable time (life-time?).

Okay, that's about it. I'd like to thank you very much for doing this interview, for sharing your insight with us and for being so patient with all my silly questions.

I guess the average reader is as silly as you! Just kidding!

Let's hope they're not...!

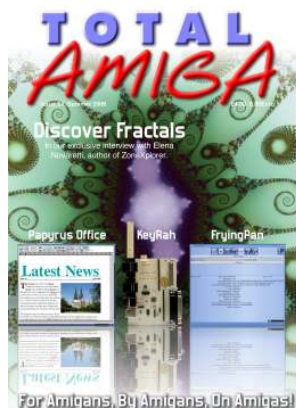
Since I'm still flabbergasted by the intricacies of the fractal realm, I think I'll leave the fractal artwork to the real artists and be on my way!

Anyone who's interested and motivated can become an expert and an artist at any time.

Lastly, anything you'd like to add?

Making war between poor men never helped. Anyone with eyes in his head can see that. Release all those nasty keys, evil jackals, and open the doors to your brothers and sisters so that they can do the same to you, and all together fight to survive.

Ciao!



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